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2. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \quad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate  $f(x)$  and find  $f'(2)$ .

(3)

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4. Joan brings a cup of hot tea into a room and places the cup on a table. At time  $t$  minutes after Joan places the cup on the table, the temperature,  $\theta^\circ\text{C}$ , of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where  $A$  and  $k$  are positive constants.

Given that the initial temperature of the tea was  $90^\circ\text{C}$ ,

- (a) find the value of  $A$ . (2)

The tea takes 5 minutes to decrease in temperature from  $90^\circ\text{C}$  to  $55^\circ\text{C}$ .

- (b) Show that  $k = \frac{1}{5} \ln 2$ . (3)

- (c) Find the rate at which the temperature of the tea is decreasing at the instant when  $t = 10$ . Give your answer, in  $^\circ\text{C}$  per minute, to 3 decimal places. (3)

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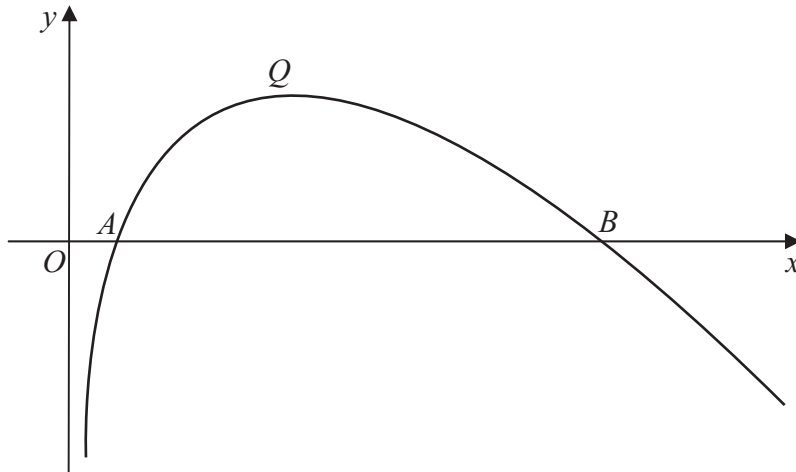
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5.



**Figure 1**

Figure 1 shows a sketch of part of the curve with equation  $y = f(x)$ , where

$$f(x) = (8 - x) \ln x, \quad x > 0$$

The curve cuts the  $x$ -axis at the points  $A$  and  $B$  and has a maximum turning point at  $Q$ , as shown in Figure 1.

(a) Write down the coordinates of  $A$  and the coordinates of  $B$ . (2)

(b) Find  $f'(x)$ . (3)

(c) Show that the  $x$ -coordinate of  $Q$  lies between 3.5 and 3.6 (2)

(d) Show that the  $x$ -coordinate of  $Q$  is the solution of 
$$x = \frac{8}{1 + \ln x}$$
 (3)

To find an approximation for the  $x$ -coordinate of  $Q$ , the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking  $x_0 = 3.55$ , find the values of  $x_1$ ,  $x_2$  and  $x_3$ .  
Give your answers to 3 decimal places. (3)







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6. The function  $f$  is defined by

$$f: x \mapsto \frac{3 - 2x}{x - 5}, \quad x \in \mathbb{R}, \quad x \neq 5$$

(a) Find  $f^{-1}(x)$ .

(3)

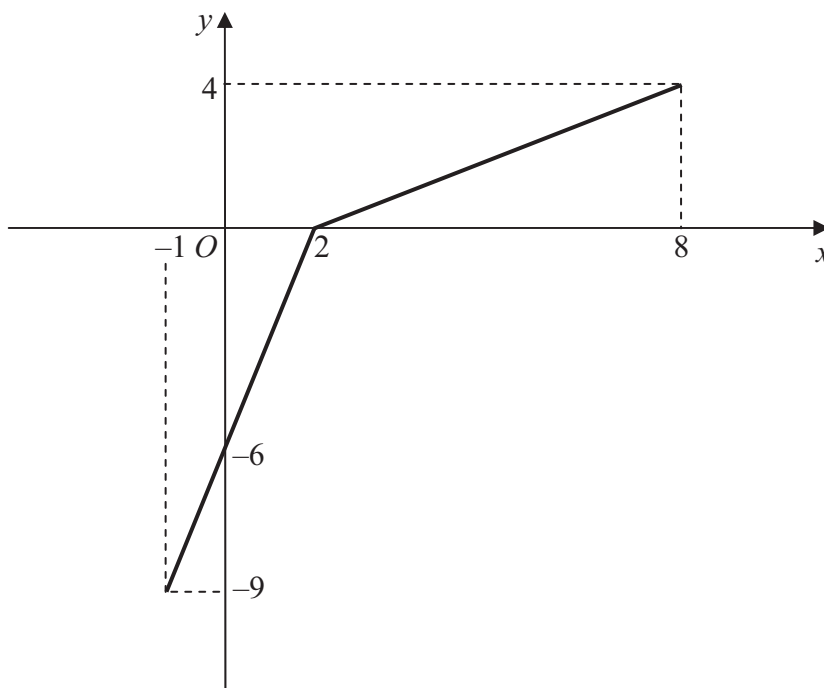


Figure 2

The function  $g$  has domain  $-1 \leq x \leq 8$ , and is linear from  $(-1, -9)$  to  $(2, 0)$  and from  $(2, 0)$  to  $(8, 4)$ . Figure 2 shows a sketch of the graph of  $y = g(x)$ .

(b) Write down the range of  $g$ .

(1)

(c) Find  $gg(2)$ .

(2)

(d) Find  $fg(8)$ .

(2)

(e) On separate diagrams, sketch the graph with equation

(i)  $y = |g(x)|,$

(ii)  $y = g^{-1}(x).$

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function  $g^{-1}$ .

(1)





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7. The curve  $C$  has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to  $C$  at the point on  $C$  where  $x = \frac{\pi}{2}$ .

Write your answer in the form  $y = ax + b$ , where  $a$  and  $b$  are exact constants.

(4)

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8. (a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that  $\frac{d}{dx}(\sec x) = \sec x \tan x$ .

**(3)**

Given that

$$x = \sec 2y$$

(b) find  $\frac{dx}{dy}$  in terms of  $y$ .

**(2)**

(c) Hence find  $\frac{dy}{dx}$  in terms of  $x$ .

**(4)**

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**Question 8 continued**

Lined area for student response

**Q8**

**(Total 9 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

